

# Trial of Galileon gravity by cosmological expansion and growth observations

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Galileon gravity is a robust theoretical alternative to general relativity with a cosmological constant for explaining cosmic acceleration, with interesting properties such as having second order field equations and a shift symmetry. While either its predictions for the cosmic expansion or growth histories can approach standard  $\Lambda$ CDM, we demonstrate the incompatibility of both doing so simultaneously. Already current observational constraints can severely disfavor an entire class of Galileon gravity models that do not couple directly to matter, ruling them out as an alternative to  $\Lambda$ CDM.

## I. INTRODUCTION

General relativity is an excellent description of gravitation on all scales at which it has been tested, from the solar system to compact objects to cosmology. However within cosmology, general relativity requires a cosmological constant or some form of strongly negative pressure to explain observations of late time cosmic acceleration [1–5]. No compelling explanation exists for the magnitude of the cosmological constant (or scalar field potential), and in general such a contribution to the gravitational action should receive corrections from high energy physics.

These fine tuning and naturalness issues motivate exploration of further physics that can explain the observed acceleration and cosmic gravity, while being protected against high energy radiative corrections. One of the most successful such theories is Galileon gravity [6–8]. In the standard, cosmological constant free case this involves four terms in a Lagrangian that leads to well behaved, second order equations of motion. The Galileon field arises as a geometric object in higher dimensions and acts in 4D like a shift symmetric scalar field (dark energy), protected against radiative corrections in the absence of matter. When introducing matter, a coupling is generically expected to arise which would break the shift symmetry and spoil the radiative stability of the model. However throughout this work we concentrate solely on a Galileon scalar field that does not couple explicitly to matter (we call such a model the ‘uncoupled’ Galileon).

In this *Letter* we confront this robust theoretical alternative to general relativity with current observational data. This serves as a significant example of advancing application of cosmological data to probe gravity. One of the leading alternatives, Galileon gravity specifically has intriguing cosmological properties such as evolution from a high redshift matter dominated attractor to current acceleration and a future de Sitter attractor, and having a time varying effective Newton’s constant  $G_{\text{eff}}$ . These cosmological characteristics were investigated in detail in [9] (the existence of an asymptotic de Sitter state was first found in [10]); numerous other studies [11–18] have examined various properties of Galileon gravity. Despite its attractive theoretical properties, though, does Galileon

gravity remain viable observationally – what is the prohibitive power of current data?

We begin by an analytic discussion of the general dependencies of the effective dark energy equation of state and gravitational strength on the Galileon parameters. These will exhibit a tension between the cosmic expansion and growth trends, so we proceed numerically with a full Markov Chain Monte Carlo scan through the Galileon parameter space, comparing the theoretical predictions to the latest cosmological data sets.

## II. COSMOLOGICAL PROPERTIES OF GALILEONS

The Galileon action is that of a scalar field  $\pi$  nonlinearly and derivatively coupled to itself, and in curved spacetime to the Ricci tensor and its contractions. The action leads to field equations no higher than second order (and is hence a subset of Horndeski’s theory [19]), and is invariant under the symmetry  $\pi \rightarrow \pi + c + b_\mu x^\mu$  in the flat space limit, for constant  $c$  and  $b_\mu$ . These conditions results in four invariant combinations [7]

$$\begin{aligned}\mathcal{L}_2 &= (\nabla_\mu \pi)(\nabla^\mu \pi), & \mathcal{L}_3 &= (\Box \pi)(\nabla_\mu \pi)(\nabla^\mu \pi)/M^3 & (1) \\ \mathcal{L}_4 &= (\nabla_\mu \pi)(\nabla^\mu \pi) [2(\Box \pi)^2 - 2\pi_{;\mu\nu}\pi^{;\mu\nu} - R(\nabla_\mu \pi)(\nabla^\mu \pi)/2] / M^6 & \\ \mathcal{L}_5 &= (\nabla_\mu \pi)(\nabla^\mu \pi) [(\Box \pi)^3 - 3(\Box \pi)\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu}{}^{;\nu}\pi_{;\nu}{}^{;\rho}\pi_{;\rho}{}^{;\mu} & (3) \\ & - 6\pi_{;\mu}\pi^{;\mu\nu}\pi^{;\rho}G_{\nu\rho}] / M^9 & (4)\end{aligned}$$

where  $R$  is the Ricci scalar,  $G_{\nu\rho}$  the Einstein tensor, and  $M^3 = M_{\text{pl}} H_0^2$  with  $M_{\text{pl}}$  the Planck mass and  $H_0$  the Hubble constant. The full action is then

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2 R}{2} - \frac{1}{2} \sum_{i=2}^5 c_i \mathcal{L}_i - \mathcal{L}_m \right] \quad (5)$$

where  $c_{2-5}$  are arbitrary dimensionless constants,  $g$  is the determinant of the metric, and  $\mathcal{L}_m$  is the matter Lagrangian that contains no  $\pi$  dependence in this work. Generalization of the coefficients to be functions of the field and its canonical kinetic term is possible [20–22], but we consider the standard Galileon case where the coefficients are constants.

First, let us explore the broad effects of the Galileon parameters. At high redshift, as discussed by [9], the effective dark energy equation of state  $w(z)$  follows tracker trajectories given by the background equation of state, i.e. radiation or matter domination, and so is independent of the parameters. The strength of the gravitational coupling  $G_{\text{eff}}$ , however, deviates from Newton's constant  $G_N$  by an amount proportional to the dark energy density at the time,  $\Omega_{de}$ . Thus a key early parameter is the initial dark energy density  $\rho_{\pi,i}(c_n, H_i, x_i)$ , where  $x = d(\pi/M_{pl})/d\ln(1+z)$  is the field velocity and  $z$  the cosmic redshift.

Analytically, increasing  $\rho_{\pi,i}$  increases the gravitational strength. A substantial increase in the gravitational strength would enhance the growth of structure, even at later times, enough to make it discrepant with observations. So we expect that growth constraints would favor low values of  $\rho_{\pi,i}$ , keeping  $G_{\text{eff}} \approx G_N$  for the matter dominated era.

[More technically, since  $\rho_{\pi,i}$  is a function of  $c_n$ , this favors a certain region of the Galileon parameter space. At high redshift the  $c_5$  term typically dominates over the others, by factors of  $H^2 x \gg 1$ . For increasingly larger initial densities (and larger  $c_5$ ), it takes longer for the other  $c_n$  terms to give comparable contributions. Since the moderate redshift ( $z \approx 10$ ) peak in the gravitational strength  $G_{\text{eff}}$  noted in [9] occurs due to interplay and partial cancellation between the terms, then higher values of  $\rho_{\pi,i}$  shift the peak to later times. Once the cancellation passes, the peak in the gravitational strength often gives way to a period around  $z \approx 3$  where  $G_{\text{eff}} \approx G_N$  is restored. Finally, the growth of the dark energy density fraction  $\Omega_{de}$  moves  $G_{\text{eff}}$  instead toward its late time de Sitter attractor behavior, which is independent of  $\rho_{\pi,i}$ . The basic point, however, is that increasing  $\rho_{\pi,i}$  tends to amplify the deviation from Einstein gravity, particularly at  $z \approx 3 - 10$ .]

The opposite dependence is true, however, for the expansion constraints. If we start with a low  $\rho_{\pi,i}$ , then due to the approximate tracking behavior of  $\rho_\pi$  during matter domination *and* the fact that we still need to arrive today at  $\Omega_{de,0} \approx 0.7$ , one requires a more extreme evolution in  $w(z)$  near the present. Analytically, to catch up the dark energy density to the present value one must have highly negative values of  $w(z)$  at  $0 \lesssim z \lesssim 2$ . Thus, low  $\rho_{\pi,i}$  leads to strong spikes in  $w(z)$ . This shifts the distance-redshift relation from the observed, near- $\Lambda$ CDM behavior, both at low redshift and for the integrated distance to the CMB last scattering surface.

Thus one has a simple analytic picture: Galileon gravity is caught between the Scylla of high initial density (and the related region of  $c_n$  parameter space) pulling  $G_{\text{eff}}$  and growth unviably up, and the Charybdis of low initial density pulling  $w(z)$  and distances unviably down. (In Homer's *Odyssey*, Scylla was a cliff-dwelling monster pulling sailors up from ships and Charybdis a sea monster sucking them down.)

Figure 1 illustrates this tension between the growth

history and expansion history behaviors in Galileon cosmology. The question is whether there is a safe path between the monsters. This requires exact numerical computation, scanning over the Galileon parameter space with Markov Chain Monte Carlo techniques.

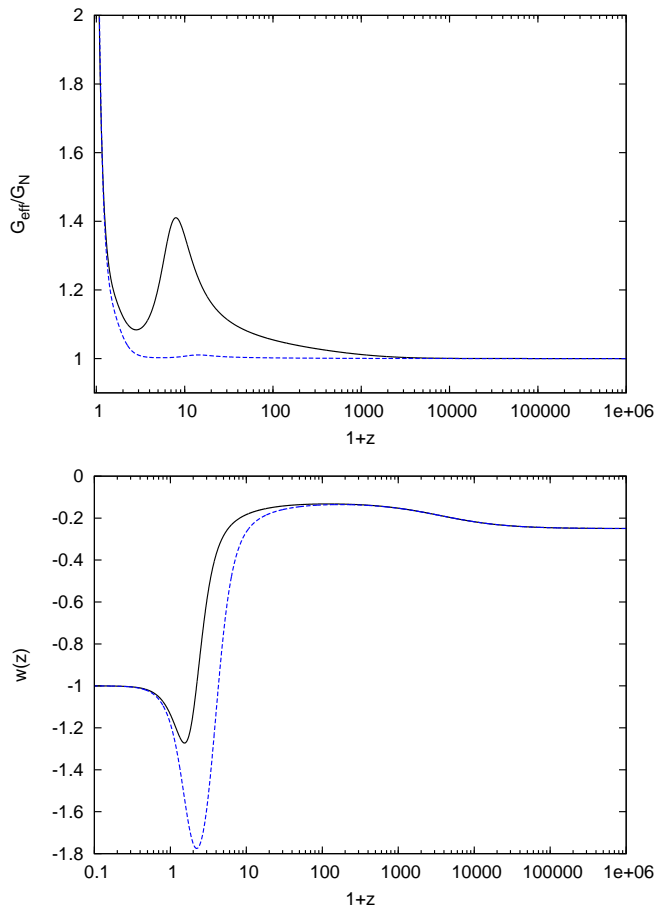


FIG. 1. Gravitational coupling deviation  $G_{\text{eff}}/G_N$  and effective equation of state  $w$  are shown for examples with high redshift initial conditions  $\rho_{\pi,i} = 10^{-6}\rho_{m,i}$  (dashed) and  $\rho_{\pi,i} = 3 \times 10^{-5}\rho_{m,i}$  (solid). Note that adjusting  $\rho_{\pi,i}$  to lessen deviations in gravity increases the deviations in equation of state, and vice versa.

### III. COSMOLOGICAL CONSTRAINTS

We first emphasize that when using only distance data constraints, for example those arising from the cosmic microwave background (CMB), baryon acoustic oscillation (BAO) and supernovae, we can find an acceptable fit, with a maximum likelihood comparable to  $\Lambda$ CDM (this was also found by [11]). Similarly, by using only growth data one can also find a viable parameter region (cf. the low initial density curve in Fig. 1). However, these two regions may be disjoint and the tension within the combined data constraints forces even the best fit to have a poor joint likelihood.

The second issue is that certain parts of parameter space are restricted theoretically due to ghosts or instabilities, as discussed in [9]. Indeed the best fit regions seem to tend to lie close to these because the best fits take advantage of the near cancellations between terms that can also lead to pathologies. We only apply the theoretical criteria to the past behavior of the field, that is for  $z > 0$ , since we have no reason to rule out models based on their future (observationally untested) behavior. Imposing the restrictions at all times, including the future de Sitter state, would further constrain the allowed region, increasing the tension further.

For each point in parameter space we solve for the effective dark energy equation of state ratio  $w(z)$  and the gravitational coupling  $G_{\text{eff}}(z)$  using Eqs. (18)-(19) and (24) of [9]. To stay close to quantities best constrained by data, we use  $\rho_{\pi,i}$  rather than  $x_i$  and  $\Omega_{de,0}$  rather than  $c_2$  as parameters, together with  $c_3$ ,  $c_4$ ,  $c_5$ , and  $H_0$ . We adopt a theory or inflationary prior of a spatially flat universe. We carry out the Markov Chain Monte Carlo analysis of the full likelihood surface using CosmoMC [23] as a generic sampler. The likelihood is given by the sum

$$\mathcal{L} = \mathcal{L}_{\text{CMB}} + \mathcal{L}_{\text{SN}} + \mathcal{L}_{\text{BAO}} + \mathcal{L}_{\text{growth}}. \quad (6)$$

We use the latest observational data to constrain the model. CMB data from WMAP7 is applied in the form of the covariance matrix for the shift parameter, acoustic peak multipole, and redshift of decoupling [3]. Since the Galileon model acts like the standard cosmology in the early universe these quantities basically measure the distance to last scattering and the matter density. Distances from Type Ia supernovae in the Union2.1 data compilation [24] constrain the expansion history at  $z \approx 0 - 1.4$ . Distances from the baryon acoustic oscillation feature in the galaxy distribution, measured to 6 redshifts at  $z = 0.1 - 0.7$  [25], probe a somewhat different cosmological parameter combination. For growth constraints we use measurements of the growth rate from the WiggleZ survey at four redshifts  $z = 0.2 - 0.8$  [26], and from the BOSS survey at  $z = 0.57$  [27], in the form of their Eq. (18)  $3 \times 3$  covariance matrix including expansion quantities, plus the  $E_G$  growth probe [28, 29]. Note that the main conditions needed to apply these growth data analyses to constrain a modified gravity model – that the standard  $z \gtrsim 1000$  matter transfer function and initial conditions are preserved, and that growth is scale independent over the relevant length scales – are satisfied by the Galileon case. We only use data within this scale independent range, which is below the Hubble scale  $\sim 4000\text{Mpc}$  and above the Vainshtein scale  $\sim 1\text{Mpc}$ . A thorough analysis of non-linear effects, specifically the Vainshtein screening mechanism, is beyond the scope of this paper, although we expect them to be unimportant on scales relevant to linear perturbations.

The results of the MCMC indicate the Galileon model is severely disfavored. The best fit yields a minimum  $\Delta\chi^2 = 31$  with respect to the best fit  $\Lambda\text{CDM}$  model, despite the Galileon case having 4 extra fit parameters. We

conclude that the entire parameter space of the standard Galileon theory is strongly disfavored. The CMB distance to last scattering deviates by  $\sim 3\sigma$  from the best fit  $\Lambda\text{CDM}$  case, and the individual lower redshift distances and growth predictions are similarly in moderate disagreement with  $\Lambda\text{CDM}$ . The highest impact individual constraint arises from the BOSS measurements at  $z = 0.57$ . This leverage bodes well for the impact of future redshift surveys on testing gravity on cosmic scales. The combination of all the data leads in aggregate to a poor fit.

Figure 2 shows the  $\chi^2$  surface relative to the best fit  $\Lambda\text{CDM}$  result in the plane of the CMB shift parameter  $R$  and growth rate  $f\sigma_8(z = 0.57)$ . These quantities serve as examples of cosmic expansion and growth, respectively. The  $\Delta\chi^2$  is large ( $\chi_{\text{gali}}^2 = 587.2$  to  $\chi_{\Lambda\text{CDM}}^2 = 556.5$  for the best fits) and the Galileon values are shifted considerably in attempting to fit the expansion and growth simultaneously (high yellow triangle for Galileon gravity vs low purple square for  $\Lambda\text{CDM}$ ).

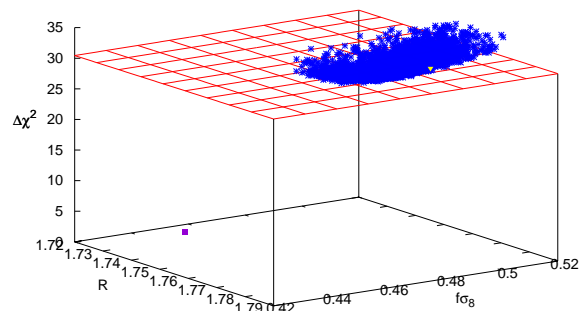


FIG. 2.  $\Delta\chi^2$  relative to the best fit  $\Lambda\text{CDM}$  model (purple square) is shown for the Galileon model as a function of the CMB shift parameter  $R$  (an example of expansion) and growth rate  $f\sigma_8(z = 0.57)$  (an example of growth, measured by BOSS [27]). Blue stars are points derived from the MCMC chains, outlining the rough paraboloid of the  $\chi^2$  surface, with the yellow triangle the best fit. No values of the Galileon parameters provide a fit with  $\Delta\chi^2 < 30.7$  (horizontal grid).

The key tension between expansion and growth will be more fully realized with more accurate data. If as an example of future data we merely change the SN data implementation to employ the Union2.1 statistics-only error matrix, rather than the full systematics matrix, the improved distance measurements exhibit the tension much more clearly, leading to a minimum  $\Delta\chi^2 = 53$ . This demonstrates how upcoming supernova surveys will also deliver substantial cosmological leverage.

The maximum likelihood values are highly stable with respect to variations in the prior ranges. The parameters  $\Omega_{m,0}$ ,  $h$ , and  $\rho_{\pi,i}$  are well constrained (with best fits for the Galileon cosmology at 0.302, 0.714, and

$\ln(\rho_{\pi,i}/\rho_{m,i}) = -11.09$ , respectively), but the coefficients  $c_3, c_4, c_5$  have strong covariance. The best fit to observational data minimizes the deviations in growth and expansion relative to  $\Lambda$ CDM, requiring a delicate balance among those Galileon coefficients. While the nominal best fits are respectively  $-2.10, -1.71, -1.77$ , there is a long, narrow region of degeneracy. (We actually also run extended ranges with logarithmic priors to ensure we are not missing a better fit. Likelihood indeed decreases for values of  $c_n$  with amplitudes much less than or greater than 1.) The degeneracy is moot, however, since the maximum likelihood is so poor.

No point in the Galileon parameter space gives a reasonable fit to current data. Moreover, the best fit, poor though it is, is achieved by balancing on the edge of a precipice: the gravitational strength diverges in the very near future. To suppress deviations in growth and expansion simultaneously the Galileon terms are forced into a highly delicate, and temporary, cancellation. (Note the divergence of  $G_{\text{eff}}$  may be ameliorated by effects beyond sub-horizon, linear perturbation theory.) Since we only applied our instability criterion to the past, where there is data, we do not rule out this model on theoretical grounds despite its Laplace instability (negative sound speed squared,  $c_s^2 < 0$ , for the dark energy perturbations) in the future. Figure 3 exhibits the gravitational strength and effective dark energy equation of state as a function of redshift for the best fit Galileon model.

#### IV. CONCLUSIONS

General relativity has passed all tests to date but lacks a clear explanation of the magnitude of the cosmological constant, or origin of dark energy, needed to account for cosmic acceleration. Two important, pressing questions are whether a sound alternative theory of gravity can explain this, and what leverage exists from current cosmological data to test such theories.

Galileon scalar fields, which have strong ties to higher dimensional gravity theories, can give rise to late time cosmic acceleration and possess well behaved, second order field equations with symmetries protecting against high energy physics renormalizations.

We analytically identify, and numerically quantify, a tension, however, between Galileon predictions for the cosmic expansion history and growth history that severely disfavors Galileon cosmology. Confronting the entire class of standard, uncoupled Galileon theory with current observations demonstrates that the predictions are a worse fit than general relativity with a cosmological constant by  $\Delta\chi^2 > 30$ . If one wanted to abandon the theory prior of spatial flatness, [11] found that adding a free parameter for curvature improved the best fit  $\chi^2$  by little more than one, and so would not have a significant effect on our conclusions.

In this work we have focussed on the case in which the scalar field is not directly coupled to ordinary matter. In

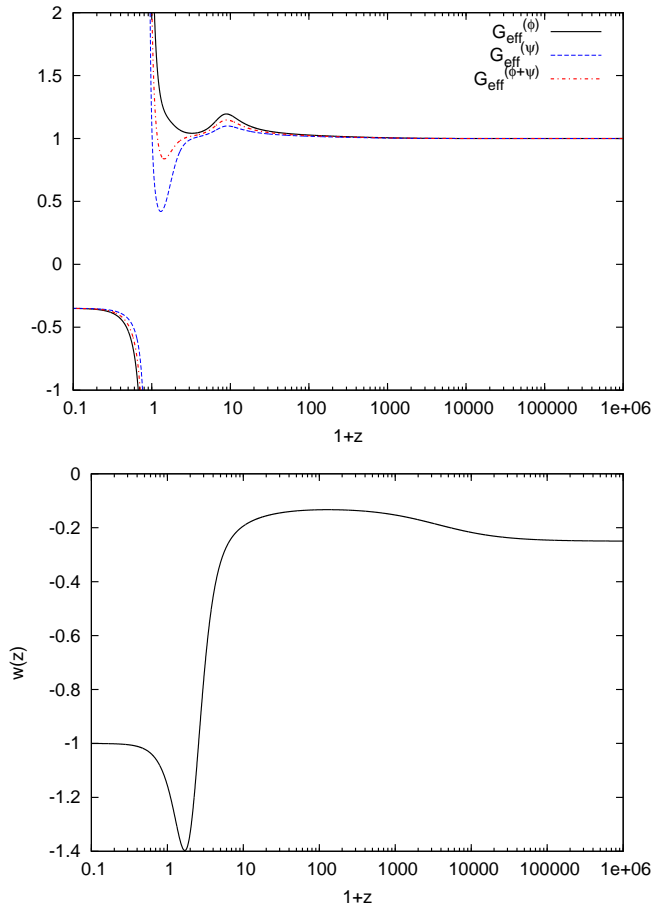


FIG. 3. Gravitational strength  $G_{\text{eff}}(z)$  and effective dark energy equation of state  $w(z)$  are plotted for the Galileon model that best fits the current data. Different  $G_{\text{eff}}$  superscripts correspond to the different modified Poisson equations in [9].

a previous paper [9] we studied the cosmological evolution in the presence of an explicit coupling. This previous work highlighted the existence of ghost and Laplace instabilities when a coupling is introduced, however an exhaustive scan of the parameter space for the coupled models remains to be undertaken (see [30] for work in this direction).

It is striking and significant that already with current data we can rule out an entire, theoretically viable class of extended gravity, one with several attractive features. We also established that forthcoming data will be able to strengthen these limits to  $\Delta\chi^2 > 50$ . More generally, the next generation of cosmological measurements will shed strong light on the distinction between modified gravity vs general relativity plus a physical dark energy, an exciting advance in understanding our universe.

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